

There is an identity involving  $\sinh x$  and  $\cosh x$  that resembles a Pythagorean identity from trigonometry.

SCORE: \_\_\_\_\_ / 7 PTS

- [a] Write that identity involving  $\sinh x$  and  $\cosh x$ . **You do NOT need to prove the identity.**

$$\cosh^2 x - \sinh^2 x = 1 \quad (1)$$

- [b] Write the identity for  $\cosh 2x$  that uses both  $\sinh x$  and  $\cosh x$  simultaneously. **You do NOT need to prove the identity.**

$$\cosh 2x = \cosh^2 x + \sinh^2 x \quad (1)$$

- [c] Use the results of [a] and [b] to find and **prove** an identity for  $\cosh 2x$  that uses only  $\sinh x$ .

$$\begin{aligned} \cosh 2x &= (1 + \sinh^2 x) + \sinh^2 x \quad (1) \\ &= 1 + 2\sinh^2 x \quad (2) \end{aligned}$$

- [d] If  $\tanh x = -\frac{2}{5}$ , find  $\sinh x$  using identities.

**You must explicitly show the use of the identities but you do NOT need to prove the identities.**

**Do NOT use inverse hyperbolic functions nor their logarithmic formulae in your solution.**

$$\begin{aligned} \operatorname{sech}^2 x &= 1 - \tanh^2 x \\ &= 1 - \frac{4}{25} \quad (1) \\ &= \frac{21}{25} \end{aligned}$$

$$\operatorname{sech} x = \frac{\sqrt{21}}{5} \quad (2)$$

SINCE  $\operatorname{sech} x > 0$  FOR ALL  $x \in \mathbb{R}$

$$\cosh x = \frac{1}{\operatorname{sech} x} = \frac{1}{\frac{\sqrt{21}}{5}} = \frac{5}{\sqrt{21}} \quad (1)$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$

$$\text{so } \sinh x = \tanh x \cosh x$$

$$= -\frac{2}{5} \cdot \frac{5}{\sqrt{21}} \quad (1)$$

$$= -\frac{2}{\sqrt{21}} = -\frac{2\sqrt{21}}{21} \quad (2)$$

Rewrite  $\operatorname{csch}(\frac{1}{2}\ln 3)$  in terms of exponential functions and simplify.

SCORE: \_\_\_\_ / 3 PTS

$$\boxed{\frac{2}{e^{\frac{1}{2}\ln 3} - e^{-\frac{1}{2}\ln 3}}} = \frac{2}{e^{\ln 3^{\frac{1}{2}}} - e^{\ln 3^{-\frac{1}{2}}}} = \boxed{\frac{2}{\sqrt{3} - \frac{1}{\sqrt{3}}}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3-1} = \boxed{\frac{2\sqrt{3}}{2}} = \boxed{\sqrt{3}}$$

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Prove that  $g(x) = \frac{1}{2} \ln \frac{1+x}{1-x}$  is the inverse of  $f(x) = \tanh x$  by simplifying  $f(g(x))$ .

SCORE: \_\_\_\_\_ / 5 PTS

You may use any identities that you found in part [1] of the Hyperbolic Functions Supplement without proving them.

$$\tanh x \left( \frac{1}{2} \ln \frac{1+x}{1-x} \right)$$

$$= \frac{e^{\frac{1}{2} \ln \frac{1+x}{1-x}} - e^{-\frac{1}{2} \ln \frac{1+x}{1-x}}}{e^{\frac{1}{2} \ln \frac{1+x}{1-x}} + e^{-\frac{1}{2} \ln \frac{1+x}{1-x}}}$$

$$= \frac{\sqrt{\frac{1+x}{1-x}} - \frac{1}{\sqrt{\frac{1+x}{1-x}}}}{\sqrt{\frac{1+x}{1-x}} + \frac{1}{\sqrt{\frac{1+x}{1-x}}}}$$

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$$\frac{\sqrt{\frac{1+x}{1-x}}}{\sqrt{\frac{1+x}{1-x}}}$$

$$= \frac{\frac{1+x}{1-x} - 1}{\frac{1+x}{1-x} + 1} \cdot \frac{1-x}{1-x}$$

$$= \frac{1+x - (1-x)}{1+x + (1-x)}$$

$$= \frac{2x}{2}$$

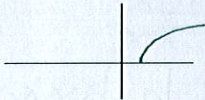
$$= x$$

Sketch the general shape and position of the following graphs.  
(Don't worry about specific  $x$  - or  $y$  - coordinates.)

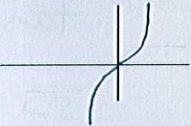
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SCORE: \_\_\_\_\_ / 3 PTS

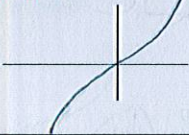
$$f(x) = \cosh^{-1} x$$



$$f(x) = \tanh^{-1} x$$



$$f(x) = \sinh x$$



Write and prove a formula for  $\cosh(x-y)$  in terms of  $\sinh x$ ,  $\sinh y$ ,  $\cosh x$  and  $\cosh y$ .

SCORE: \_\_\_\_\_ / 6 PTS

$$\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y \quad (1)$$

$$= \frac{e^x + e^{-x}}{2} \cdot \frac{e^y + e^{-y}}{2} - \frac{e^x - e^{-x}}{2} \cdot \frac{e^y - e^{-y}}{2} \quad (1)$$

$$= \frac{(e^{x+y} + e^{x-y} + e^{-x+y} + e^{-x-y})}{4} - \frac{(e^{x+y} - e^{x-y} - e^{-x+y} + e^{-x-y})}{4} \quad (1)$$

$$= \frac{2e^{x-y} + 2e^{-x+y}}{4} \quad (1)$$

$$= \frac{e^{x-y} + e^{-(x-y)}}{2} \quad (1)$$